Angular-spectrum representation of nondiffracting X waves

Juha Fagerholm,* Ari T. Friberg,† Juhani Huttunen,‡ David P. Morgan,§ and Martti M. Salomaa
Materials Physics Laboratory, Department of Technical Physics, University of Technology, Helsinki FIN-02150 Espoo, Finland
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We derive the nondiffracting X waves, first discussed within acoustics by Lu and Greenleaf [IEEE Trans. Ultrason. Ferroelec. Freq. Contr. **39**, 19 (1992)], using the general mathematical formalism based on an angular spectrum of plane waves. This serves to provide a unified treatment of not only the fundamental zeroth-order X waves of Lu and Greenleaf, but also of the lesser-known higher-order derivative X waves, first discussed here in terms of a single, universal, angular spectrum. The characteristic crossed (letter-X-like) shape and the special properties of the X waves, as well as of their angular-spectrum representation, are discussed and illustrated in detail. Asymptotically, for increasing order, the appearance of the X waves is found to transform into a triangular wedgelike waveform. [S1063-651X(96)04910-0]

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I. INTRODUCTION

The angular spectrum of plane waves (ASPW) is an intuitive and physically appealing method to study wave propagation and diffraction in problems of electrodynamics, optics, and acoustics [1]. Two types of plane-wave expansions are commonly in use: on one hand the Whittaker-type (1902) representations of source-free fields in terms of homogeneous plane waves propagating in all directions and, on the other hand, the general angular-spectrum representations effectively based on the Weyl expansion (1919) of a spherical wave [2]. The latter decomposition contains not only homogeneous but also inhomogeneous (evanescent) plane waves, and so it is applicable in aperture and near-field analyses as well.

Several classes of localized (in space and time) and nondiffracting three-dimensional (3D) wave solutions to the wave equations in free space or in uniform and isotropic media have been discovered [3–6]. The localized wave fields correspond to linear, nondispersive, wave-packet solutions, while nondiffractive waves are monochromatic beams that do not spread on propagation. The ideal properties of these fields require infinite domains and energies, but approximate solutions showing extended ranges of localization have been produced in finite apertures [7]. Coherent nondiffracting fields and their practical realizations have been studied extensively in optics [8–11] and in acoustics [6,12]. Besides high-definition metrology, these fields have potential applications, e.g., in nonlinear optics [13], particle-beam confinement [14], and inverse free-electron laser accelerators [15].

A particular subset of exact nondiffracting solutions to the free-space scalar wave equation, called the X waves, was recently put forward theoretically [6,16] and demonstrated experimentally [12] by Lu and Greenleaf. These waves may contain extended frequency bandwidths and so are pulses that on propagation remain nondiffracting in the transverse direction. The name X waves arises from the property that in a meridian plane (longitudinally, e.g., at a fixed time) the shape of the field intensity pattern resembles the letter X. The relation of the X waves to the wavelet theory was recently established via an interesting method that converts any solution of the scalar wave equation to a nondiffracting solution but with the dimensionality increased by one [16,17]. Applications of the X waves in acoustical imaging and tissue characterizations as well as in electromagnetic energy transmission have been proposed [6].

In this paper we construct the X waves and their derivatives by using the formulation in terms of an angular spectrum of plane waves. This approach provides deeper insight into the physics of the X waves, and also enlightens the relation of these waves to the general nondiffracting (monochromatic) fields that are widely used in optics [18]. It is shown that a single angular spectrum characterizes the X waves of any order and, respectively, the various time derivatives of the X waves of any order are characterized by a unique angular spectrum, each. Illustrations of the angular spectra, and of the space and time dependence of the X waves within the angular-spectrum representation are given. When the order of the X waves increases, the space-time shape of the field is discovered to approach a uniform triangular wedge.

II. THEORY OF NONDIFFRACTING X WAVES

We begin by summarizing the main results for the X waves, following the presentation by Lu and Greenleaf [6]. Consider the uniform, isotropic, 3D scalar wave equation

$$\nabla^2\Phi(\mathbf{r},t)-\frac{1}{c^2}\frac{\partial^2\Phi(\mathbf{r},t)}{\partial t^2}=0 \qquad (1)$$

for a field $\Phi(\mathbf{r},t)$ (c is the speed of the wave). Using direct substitution, it was shown by Lu and Greenleaf [6] that

*On leave of absence from Center for Scientific Computing (CSC), P.O. Box 405, FIN-02101 Espoo, Finland.

†Present address: Optisches Institut, Technische Universität Berlin, D-10623 Berlin, Germany.

‡Present address: NOKIA Telecommunications, P.O. Box 33, FIN-02601 Espoo, Finland.

§Permanent and present address: Impulse Consulting, Weston Favell, Northampton NN3 3BG, England.

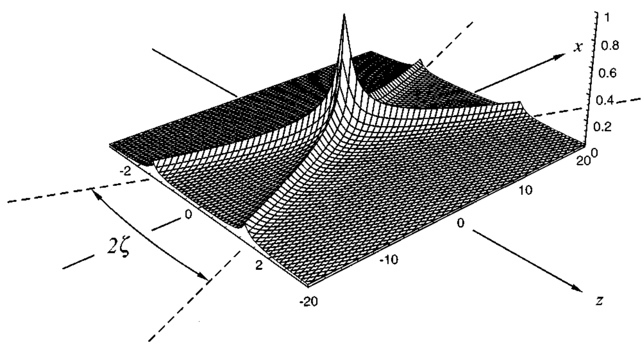

FIG. 1. Amplitude of the zeroth-order X wave, Φ_{X_0}, in the meridional xz plane. The whole X-wave pattern is rotationally symmetric about z. Parameters are chosen as follows: time $t=0$, $B(k)=a_0=0.05$ mm, $c=1.5$ mm/μs, $\zeta=4°$, and $\rho=1$ mm. The angle 2ζ is extended between the branches of the X wave.

$$\Phi_{X_n}(\mathbf{r},t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\phi_{X_n}(\mathbf{r},\omega)e^{-i\omega t}d\omega, \quad n=0,1,2,\ldots, \tag{2}$$

where

$$\phi_{X_n}(\mathbf{r},\omega)=\frac{2\pi}{c}e^{in\varphi}B\left(\frac{\omega}{c}\right)H\left(\frac{\omega}{c}\right)J_n\left(\frac{\omega}{c}\rho\sin\zeta\right)\times e^{-(\omega/c)(a_0-iz\cos\zeta)}, \tag{3}$$

is an exact solution of the wave equation in cylindrical coordinates $\mathbf{r}=(\rho\cos\varphi,\ \rho\sin\varphi,\ z)$. In Eq. (3), $B(\omega/c)$ is any (well-behaved) complex-valued function, typically a transfer function of the physical system under scrutiny, $H(\omega/c)$ is the Heaviside step function, $0<\zeta<\pi/2$ is an angle (to be defined later), $a_0>0$ is a constant, and J_n is a Bessel function of the first kind and of the nth order. The solution expressed in Eqs. (2) and (3) is called the nth-order X wave, Φ_{X_n}.

If the function $B(k)$ is a constant (and equals a_0), the X-wave solution can be written in an analytic form through Laplace transformation of the integrand. The amplitude of the fundamental (zeroth-order) X wave in a meridional plane at a fixed time ($t=0$) is illustrated in Fig. 1. [Note that the X waves in Fig. 1 (and those in Fig. 6) are special cases of the X waves represented by Eqs. (2) and (3)—those for which $B(k)=a_0=$const. These are so-called ''broadband'' X waves, Φ_{XBB_n}.] The X-like amplitude profile is rotationally symmetric with respect to the propagation axis. The wave is nondiffracting (the lateral and axial field patterns are invariant) in a coordinate system where $z-c_pt=$const, i.e., the wave propagates in the z direction at a velocity $c_p=c/\cos\zeta$. It is seen that the waves are superluminal because the velocity c_p exceeds the speed of sound or light in the medium.

The X-wave solution investigated above is of infinite extent and it has a divergent total energy, which are both characteristic features of exact, nondiffracting, solutions. Practical X waves, which can be generated with the help of physical devices, are nearly nondiffracting within a finite depth of field. The X waves produced by radiators of finite apertures and with different transfer functions $B(k)$ have been evaluated [6] using the Rayleigh-Sommerfeld diffraction theory.

Relation of nondiffracting waves to wavelets has also been discussed by Lu et al. [16]. They have shown that an $(n-1)$-dimensional wavelet solution may be transformed to a nondiffracting n-dimensional solution which, for $n=3$, is proportional to the second time derivative of the fundamental X wave.

III. CONSTRUCTION OF X WAVES IN THE ANGULAR-SPECTRUM REPRESENTATION

The angular-spectrum decomposition is readily derived from the scalar wave equation, and the representation is well established for nondiffracting fields [9]. Here we first present the main results of the general formulation and then specifically apply it to the X waves. Many physical properties of the X waves, such as the superluminous feature of conic waves, become obvious in the framework of the angular-spectrum representation. In particular, the angular-spectrum decomposition is exactly the construction within which an $(n-1)$-dimensional diffractive solution implies, through a coordinate substitution, an n-dimensional nondiffracting solution. These interesting connections will be expounded in detail within a separate publication.

A. Angular spectrum of nondiffracting waves

The wave equation, Eq. (1), for a field

$$\Phi(\mathbf{r},t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\phi(\mathbf{r},\omega)\exp(-i\omega t)d\omega \tag{4}$$

is reduced to the homogeneous Helmholtz equation

$$\nabla^2\phi(\mathbf{r},\omega)+k^2\phi(\mathbf{r},\omega)=0, \tag{5}$$

where the wave number is defined as $k=\omega/c$. The monochromatic solutions of the Helmholtz equation may be expressed in terms of the angular-spectrum decomposition

$$\phi(\mathbf{r},\omega)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}a(p,q,\omega)e^{ik(px+qy+m|z|)}dp\,dq, \tag{6}$$

where $a(p,q,\omega)$ is the angular spectrum and the mutual relationship obeyed between the triple p, q, and m is given by

$$m=\begin{cases}\sqrt{1-p^2-q^2} & \text{for } p^2+q^2\leqslant 1,\\ i\sqrt{p^2+q^2-1} & \text{for } p^2+q^2>1\,.\end{cases} \tag{7}$$

These terms represent homogeneous plane waves and exponentially decaying evanescent waves, respectively. The angular spectrum $a(p,q,\omega)$ can be calculated from the value of $\phi(\mathbf{r},\omega)$ at $z=0$, i.e.,

$$a(p,q,\omega)=\left(\frac{k}{2\pi}\right)^2\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\phi(x,y,z=0,\omega)\times e^{-ik(px+qy)}dx\,dy. \tag{8}$$

The nondiffracting relation for monochromatic fields may be defined as

$$|\phi_{\rm nd}(\mathbf{r},\omega)|^2=|\phi_{\rm nd}(x,y,z,\omega)|^2=|\phi_{\rm nd}(x,y,z=0,\omega)|^2, \tag{9}$$

which implies that the lateral intensity distribution of a nondiffracting field does not vary on propagation along the z direction. A well-known solution satisfying this condition can be derived provided that we consider source-free fields for $z\geqslant 0$ and change the rectangular coordinates in the angular-spectrum representation [Eq. (6)] to the spherical ones. (In these coordinates, α may generally be complex; however, α is real for free fields. Here we do not discuss the consequences of this; see, for example Ref. [19].) Then $\mathbf{s}=(p,q,m)=(\sin\alpha\cos\beta,\ \sin\alpha\sin\beta,\ \cos\alpha)$ and the differentials transform as $dp\,dq=\sin\alpha\cos\alpha\ d\alpha\ d\beta$. Thus the field assumes the form

$$\phi(\mathbf{r},\omega)=\int_0^{\pi/2}\int_0^{2\pi}F(\alpha,\beta,\omega)e^{ik\mathbf{s}\cdot\mathbf{r}}\sin\alpha\ \cos\alpha\ d\alpha\ d\beta, \tag{10}$$

where we have denoted

$$F(\alpha,\beta,\omega)=a(p,q,\omega). \tag{11}$$

We consider angular spectra of the type

$$F(\alpha,\beta,\omega)=A(\beta,\omega)\frac{\delta(\alpha-\alpha_0)}{|\sin\alpha_0|\cos\alpha_0}, \tag{12}$$

where $A(\beta,\omega)$ is an arbitrary function, δ is the Dirac delta function, and $\alpha_0<\pi/2$. This corresponds to the field

$$\phi_{\rm nd}(\mathbf{r},\omega)=e^{ikz\ \cos\alpha_0}\int_0^{2\pi}A(\beta,\omega)\times\exp[ik\ \sin\alpha_0(x\ \cos\beta+y\ \sin\beta)]d\beta, \tag{13}$$

which satisfies the nondiffracting condition. The field consists of plane waves with their wave vectors on the surface of a cone, in which the top angle is $2\alpha_0$; see Fig. 2. The superluminal property of the conic waves is obvious in this representation because the intersection of the wave fronts is seen to propagate at the speed $c_p=c/\cos\alpha_0$.

If we further change the x and y coordinates into the cylindrical coordinate representation through the relation $x\ \cos\beta+y\sin\beta=\rho\cos(\beta-\varphi)$, we get the following expression for the field

$$\phi_{\rm nd}(\mathbf{r},\omega)=e^{ikz\ \cos\alpha_0}\int_0^{2\pi}A(\beta,\omega)\times\exp[ik\rho\ \sin\alpha_0\cos(\beta-\varphi)]d\beta. \tag{14}$$

This is a general nondiffracting solution, including the rotationally symmetric and nonsymmetric fields with respect to the z axis. This result has been investigated extensively (see, e.g., Refs. [8,9]).

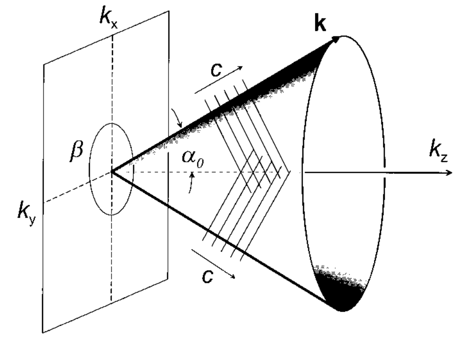

FIG. 2. Wave vectors $\mathbf{k}$ of a nondiffracting wave lie on a cone with the top angle $2\alpha_0$ in momentum space. The angular variable β has values in the range $[0,2\pi]$. The plane waves propagate at the speed c along the cone but their interference pattern has a superluminous velocity $c_p=c/\cos\alpha_0$ in the z direction.

Now suppose that $A(\beta,\omega)$ is sufficiently well-behaved, such that it can be expanded in a Fourier series (with the period 2π):

$$A(\beta,\omega)=\sum_{n=-\infty}^{\infty}a_n(\omega)e^{in\beta}=\sum_{n=-\infty}^{\infty}a_n(\omega)e^{in\varphi}e^{in(\beta-\varphi)}. \tag{15}$$

Inserting Eq. (15) into Eq. (14) gives

$$\phi_{\rm nd}(\mathbf{r},\omega)=e^{ikz\ \cos\alpha_0}\sum_{n=-\infty}^{\infty}a_n(\omega)\exp(in\varphi)\times\int_0^{2\pi}\exp[in(\beta-\varphi)]\times\exp[ik\rho\ \sin\alpha_0\cos(\beta-\varphi)]d(\beta-\varphi). \tag{16}$$

Moreover, using a known identity of Bessel functions, we find

$$\phi_{\rm nd}(\mathbf{r},\omega)=e^{ikz\ \cos\alpha_0}\sum_{n=-\infty}^{\infty}c_n(\varphi,\omega)J_n(k\rho\ \sin\alpha_0), \tag{17}$$

where we have introduced

$$c_n(\varphi,\omega)=2\pi\ \exp\left[in\left(\varphi+\frac{\pi}{2}\right)\right]a_n(\omega). \tag{18}$$

The nondiffracting solution consists of a sum of Bessel functions with varying order n multiplied by the complex coefficients $c_n(\varphi,\omega)$, in which the phase only depends on the azimuthal angle φ. This implies a cylindrically symmetric lateral intensity distribution for the individual terms, but not for the total field. Note that each term in the sum represents a nondiffracting field as well.

B. Construction of X waves

We establish the relation between the nondiffracting expression in Eq. (17) and the spectrum of the X waves in Eq.

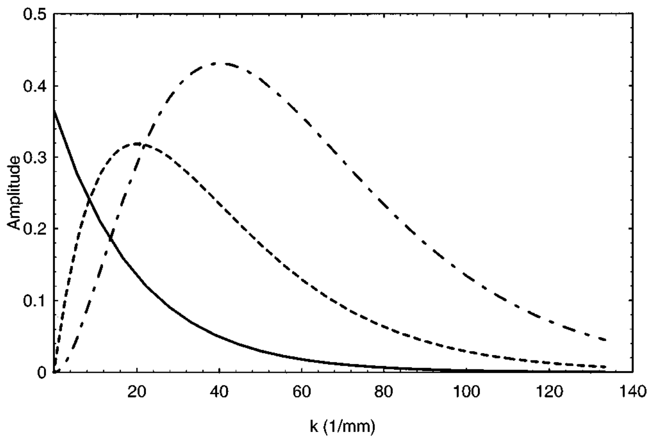

FIG. 3. Normalized amplitude of the coefficient for the angular spectrum $A_n^{(m)}(\beta,\omega)$ in Eq. (24) with $m=0$ (solid curve), $m=1$ (dashed curve) and $m=2$ (dashed-dotted curve). The normalization factor chosen is the square root of the integrated intensity of the X-wave spectrum. The parameters employed are $B(k)=a_0=0.05$ mm, $c=1.5$ mm/μs, $\zeta=4°$, and $\rho=1$ mm.

(3), and derive the angular spectrum for the X waves. If we choose $c_n=(2\pi/c)e^{in\varphi}B(k)H(k)e^{-ka_0}$ in Eq. (17), we find

$$\phi(\mathbf{r},\omega)=\frac{2\pi}{c}B(k)H(k)e^{-k(a_0-iz\cos\alpha_0)}\times\sum_{n=-\infty}^{\infty}e^{in\varphi}J_n(k\rho\,\sin\alpha_0). \tag{19}$$

Any term in the sum in Eq. (19),

$$\phi_n(\mathbf{r},\omega)=\frac{2\pi}{c}e^{in\varphi}B(k)H(k)J_n(k\rho\,\sin\alpha_0)e^{-k(a_0-iz\,\cos\alpha_0)}, \tag{20}$$

coincides with the spectrum $\phi_{X_n}(\mathbf{r},\omega)$ of the X waves [Eq. (3)] if we choose $\alpha_0=\zeta$. Note that these angles have exactly the same value, although their original physical interpretations are quite different. The angle ζ is associated with the half-angle between the orientation of the X branches in Fig. 1, while the angle α_0 is half the top angle of the cone of wave vectors in Fig. 2.

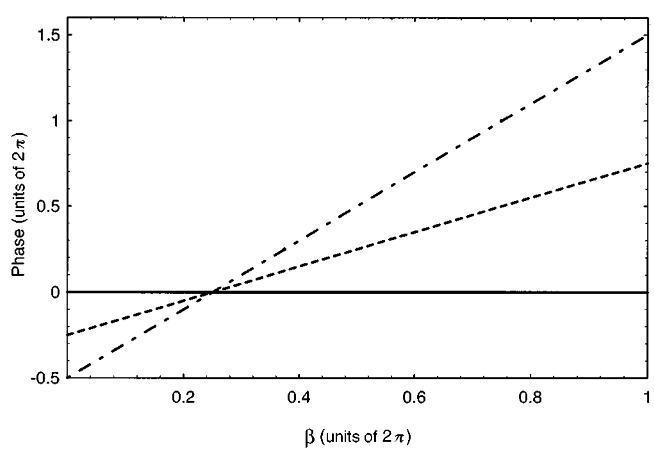

FIG. 4. Phase of the coefficient for the angular spectrum $A_n^{(m)}(\beta,\omega)$ in Eq. (24) for $m=0$ and $n=0$ (solid curve), $n=1$ (dashed curve), and $n=2$ (dashed-dotted curve).

The angular spectrum of the X waves can now be found by using the $c_n(\varphi,\omega)$ introduced in Eqs. (18), (15), (12), and (11):

$$F_n(\alpha,\beta,\omega)=A_n(\beta,\omega)\frac{\delta(\alpha-\zeta)}{|\sin\zeta|\cos\zeta}, \tag{21}$$

where

$$A_n(\beta,\omega)=\frac{1}{c}B(k)H(k)e^{-ka_0}\exp\left[in\left(\beta-\frac{\pi}{2}\right)\right]. \tag{22}$$

The expression in Eq. (21) is of the same type as the ordinary nondiffracting solution in Eq. (12) with the wave vectors on the surface of the cone: $\alpha=\zeta$.

In view of Eq. (4), the spectrum of the mth-order derivative (with respect to time) of the X wave is the fundamental X-wave spectrum multiplied by $(-i\omega)^m$, and hence the derivative is also a nondiffracting beam. Consequently, the angular spectrum of the X waves and their derivatives may be written in a unique, ''universal,'' form:

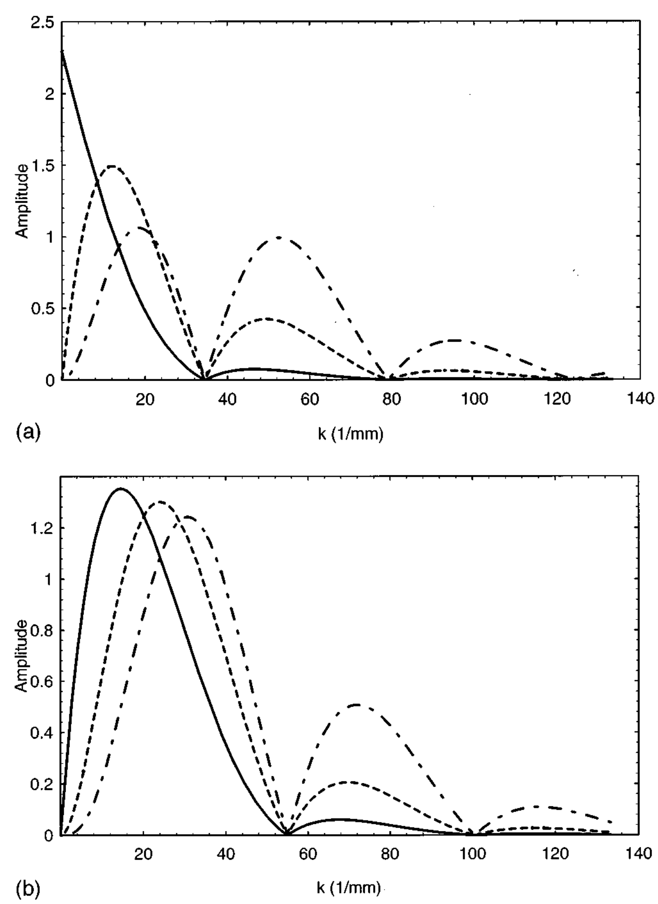

FIG. 5. (a) Normalized amplitude of the X-wave spectrum for $n=0$ and $m=0$ (solid curve), $m=1$ (dashed curve), and $m=2$ (dashed-dotted curve). (b) Same as (a) but now for $n=1$. The normalization factor is the square root of the integrated intensity of the X-wave spectrum. The parameters employed are $B(k)=a_0=0.05$ mm, $c=1.5$ mm/μs, $\zeta=4°$, and $\rho=1$ mm.

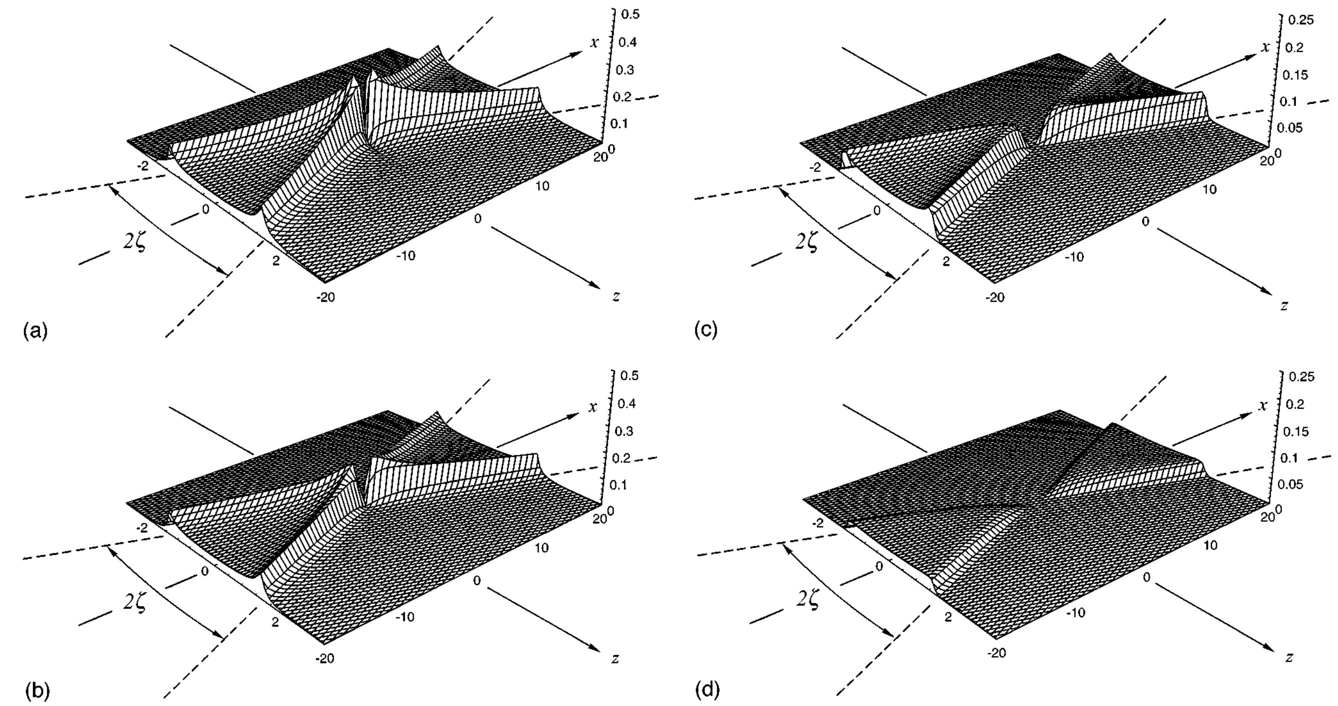

FIG. 6. Amplitude of the X waves, Φ_{X_n}, for higher n-values (cf. Fig. 1 for the fundamental $n=0$ X-wave mode), in a meridian xz plane at time $t=0$: (a) $n=1$, (b) $n=2$, (c) $n=6$, and (d) $n=15$. Parameters used: $B(k)=a_0=0.05$ mm, $c=1.5$ mm/μs, $\zeta=4°$, and $\rho=1$ mm.

$$F_n^{(m)}(\alpha,\beta,\omega)=A_n^{(m)}(\beta,\omega)\,\frac{\delta(\alpha-\zeta)}{|\sin\zeta|\cos\zeta}, \tag{23}$$

where

$$A_n^{(m)}(\beta,\omega)=k^m c^{m-1}B(k)H(k)e^{-ka_0} \times\exp\left\{i\left[n\beta-(n+m)\frac{\pi}{2}\right]\right\}. \tag{24}$$

In the angular-spectrum representation we may construct different types of X waves [including the Φ_{X_n} in Eq. (2)] by choosing the coefficients $A_n^{(m)}(\beta,\omega)$ according to Eq. (24).

IV. ILLUSTRATIONS

We find it quite instructive to visualize in detail the angular spectra, the time-domain spectra, and the higher-order forms of the X waves, and to investigate the properties of the X waves on the basis of the relations derived in the previous section.

A. Angular spectra

Characteristically to any nondiffracting waves, the wave vectors for the X waves lie on the surface of a cone, whose top angle is 2ζ (cf. Fig. 2). In a situation where all the other parameters are fixed, the magnitudes of the wave vectors have a minimum for the angle $\zeta=\pi/4$ and increase sinusoidally for decreasing and increasing angles, cf. Eq. (21).

Note, in particular, that the amplitude of the angular spectrum $F_n^{(m)}(\alpha,\beta,\omega)$ is independent of the order n (for fixed m). Especially, the amplitude is universal for all solutions Φ_{X_n}. Instead, the amplitude changes with the order m of the derivative (for fixed n). The amplitude curves are shown in Fig. 3 as functions of k.

The phase of the angular spectrum $F_n^{(m)}(\alpha,\beta,\omega)$ is independent of k but the linear phase change with respect to the azimuthal angle β depends on the order n of the wave. The phase curves are illustrated in Fig. 4 for the case $m=0$.

B. Time-domain spectra

The functional form of the amplitude of the X-wave spectra is determined as the amplitude of the angular spectrum multiplied by the appropriate nth-order Bessel function [see Eqs. (20) and (24)]. The amplitudes of the spectra for the zeroth-order X wave and its first and second derivatives as well as for the first-order X wave and its first two derivatives are shown in Fig. 5.

The spectra in Fig. 5(a) resemble (although the parameters chosen are different) the spectra plotted in Ref. [6] at low frequencies. To supplement the results of Lu and Greenleaf [16], we have in addition shown that the sidelobes are present in the spectra at higher frequencies.

C. Dark-beam X waves

In addition to the fundamental nondiffracting wave corresponding to the zeroth-order Bessel function, there is also a great interest in the higher-order waves in several branches of physics [9]. In particular, the first-order wave is of main interest because the intensity vanishes at the center of the beam. This so-called ''dark beam'' with a small and well-defined dark central spot has applications, for example, in precision alignment.

Lu and Greenleaf consider mainly the fundamental X wave [see Fig. 1] in their publications. Here we are interested to illustrate and analyze the special properties of higher-order X waves within the angular-spectrum representation. The amplitude distributions for the X waves of the orders 1, 2, 6, and 15 are plotted in Fig. 6.

These waves have a (nearly) dark central spot which is characteristic to all of the higher-order Bessel beams. The intensity spreads gradually away from the z axis as the order n of the wave increases. The branches of letter X are clearly visible when n is 1 or 2, but the intensity distribution finally seems to approach a triangular wedge shape, where the intensity becomes evenly distributed between the propagation fronts of the X. In every case, however, the intensity is concentrated within the area defined by the X branches.

V. DISCUSSION AND CONCLUSIONS

We have shown that the angular-spectrum representation of plane waves provides a unified treatment for the nondiffracting X waves, first discovered by Lu and Greenleaf, and also for the temporal derivatives of the X waves, which likewise are nondiffracting solutions of the same wave equation. For $n>0$, these are so-called ''dark beams.''

Recently, the X waves have attracted wide interest both in acoustics and optics; for example, the X waves are being applied within novel methods of designing femtosecond light fields, X pulses, in such a way that they would maintain their extended longitudinal and lateral localizations during propagation into considerable depths in a given dispersive medium [20].

In particular, we have shown that a single, ''universal'' angular-spectrum representation serves to produce both the fundamental X wave and the higher-order derivative X waves as well. We have examined the distribution of the X-wave amplitude in the meridian plane and we have further shown that for increasing order the X waves tend to approach a triangular wedge shape.

Our present approach facilitates the treatment of X waves on the same general mathematical footing as that for nondiffracting waves in other branches of physics, such as electrodynamics and optics. We consider these connections useful for further theoretical and experimental investigations of the X waves.

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